Finite sample size optimality of GLR tests

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Outline

- Hypothesis testing and GLRT
- Randomized tests
  - Classical hypothesis testing with randomized tests
  - Alternative implementation of randomized tests
- A combined hypothesis testing/isolation problem – Optimality of GLRT
- Randomized estimators
- A combined hypothesis testing/estimation problem – Optimum GLR-type tests

Hypothesis testing and GLRT

Binary hypothesis testing (fixed sample size N):
Collection of observations $X=[x_1,\ldots,x_N]$

$$\mathbb{H}_0\colon X \sim f_0(X);$$
$$\mathbb{H}_1\colon X \sim f_1(X);$$

Decide between $\mathbb{H}_0$ and $\mathbb{H}_1$

Likelihood Ratio Rest (LRT) $$\frac{f_1(X)}{f_0(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

Optimum according to different criteria:

Neyman-Pearson, Bayesian, Min-Max, Equalized error probabilities.

Composite binary hypothesis testing

Observations $X=[x_1,\ldots,x_N]$

$$
\begin{aligned}
\mathbb{H}_0 : X &\sim f_{01}(X), && \pi_{01}\\
&\sim f_{02}(X), && \pi_{02}\\
&\vdots\\
&\sim f_{0K_0}(X), && \pi_{0K_0}\\
\mathbb{H}_1 : X &\sim f_{11}(X), && \pi_{11}\\
&\sim f_{12}(X), && \pi_{12}\\
&\vdots\\
&\sim f_{1K_1}(X), && \pi_{1K_1}
\end{aligned}
$$

One subcase responsible for the data

If we knew before hand the two possible subcases, for example: f_{0i} and f_{1j} then

$$\frac{f_{1j}(X)}{f_{0i}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

When this knowledge is not possible:

$$\frac{f_1(X)}{f_0(X)} = \frac{\pi_{11} f_{11}(X) + \cdots + \pi_{1K_1} f_{1K_1}(X)}{\pi_{01} f_{01}(X) + \cdots + \pi_{0K_0} f_{0K_0}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

If in addition to detection, we like to **isolate** the subcase responsible for the data X

GLRT

$$\frac{f_{1\hat{j}}(X)}{f_{0\hat{i}}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

$$\hat{i} = \arg \max_{1 \le i \le K_0} f_{0i}(X)$$

$$\hat{j} = \arg \max_{1 \le j \le K_1} f_{1j}(X)$$

ML isolation

Composite binary hypothesis testing (cond.)

Observations $X=[x_1,\ldots,x_N]$

$$\mathbb{H}_0 : X \sim f_0(X|\theta_0), \quad \pi_0(\theta_0)$$

$$\mathbb{H}_1 : X \sim f_1(X|\theta_1), \quad \pi_1(\theta_1)$$

Only detection

$$\frac{f_1(X)}{f_0(X)} = \frac{\int \pi_1(\theta_1) f_1(X|\theta_1)\, d\theta_1}{\int \pi_0(\theta_0) f_0(X|\theta_0)\, d\theta_0} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

If we like, in addition to detection to **estimate** the parameter vector θ_i

GLRT

$$\frac{f_1(X|\hat{\theta}_1)}{f_0(X|\hat{\theta}_0)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

$$\hat{\theta}_0 = \arg\max_{\theta_0} f_0(X|\theta_0)$$

$$\hat{\theta}_1 = \arg\max_{\theta_1} f_1(X|\theta_1)$$

ML estimate

Optimality of GLRT

Wald (1943)

Le Cam (1953)

Lehmann (1959)

Hoeffding (1965)

Neyman (1965)

⋮

Asymptotic optimality under **regularity conditions**. As the number of samples increases without limit, performance of GLRT approaches the optimum test (with true parameters considered known).

Randomized tests

Binary hypothesis testing (fixed sample size N):
Collection of observations $X=[x_1,\ldots,x_N]$

$$\mathbb{H}_0\colon X \sim f_0(X);$$
$$\mathbb{H}_1\colon X \sim f_1(X);$$

Decide between $\mathbb{H}_0$ and $\mathbb{H}_1$

Deterministic tests

$X \in \mathbb{R}^N$

Two complementary sets in $\mathbb{R}^N$: A_0, A_1 with $A_1^c = A_0$

When $X \in A_i$ decide for $\mathbb{H}_i$.

Randomized tests

Two complementary probabilities: $\delta_0(X), \delta_1(X)$ with $\delta_0(X)+\delta_1(X)=1$.

In a **random game** select $\mathbb{H}_i$ with probability $\delta_i(X)$.

Randomized more general than Deterministic

$$\delta_i(X) = 1_{A_i}(X)$$

When $X \in A_1$ then we decide with **probability 1** for $\mathbb{H}_1$ and with probability 0 for $\mathbb{H}_0$ (the equivalent of a deterministic decision).

Neyman-Pearson:

$$\max_{\delta_0,\delta_1} \mathbb{P}[d=1|\mathbb{H}_1]; \quad \text{subject to} \quad \mathbb{P}[d=1|\mathbb{H}_0] \le \alpha$$

$$\int \delta_1(X) f_1(X)\, dX \quad - \quad \lambda \int \delta_1(X) f_0(X)\, dX$$

$$= \int \delta_1(X)[f_1(X) - \lambda f_0(X)]\, dX$$

$$\le \int \max\{f_1(X) - \lambda f_0(X), 0\}\, dX$$

$$\delta_1(X) = \begin{cases} 1 & f_1(X) - \lambda f_0(X) > 0 \\ 0 & f_1(X) - \lambda f_0(X) < 0 \\ \gamma & f_1(X) - \lambda f_0(X) = 0. \end{cases}$$

$$\delta_0(X) = 1 - \delta_1(X) = \begin{cases} 0 & f_1(X) - \lambda f_0(X) > 0 \\ 1 & f_1(X) - \lambda f_0(X) < 0 \\ 1-\gamma & f_1(X) - \lambda f_0(X) = 0. \end{cases}$$

Likelihood Ratio Rest (LRT)

$$\frac{f_1(X)}{f_0(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

Composite binary hypothesis testing

Observations $X=[x_1,\ldots,x_N]$

$$
\begin{array}{lcll}
\mathbb{H}_0 : X & \sim & f_{01}(X), & \pi_{01} \quad \delta_{01}(X) \\
 & \sim & f_{02}(X), & \pi_{02} \quad \delta_{02}(X) \\
 & \vdots & & \vdots \\
 & \sim & f_{0K_0}(X), & \pi_{0K_0} \quad \delta_{0K_0}(X) \\
 & & & \\
\mathbb{H}_1 : X & \sim & f_{11}(X), & \pi_{11} \quad \delta_{11}(X) \\
 & \sim & f_{12}(X), & \pi_{12} \quad \delta_{12}(X) \\
 & \vdots & & \vdots \\
 & \sim & f_{1K_1}(X), & \pi_{1K_1} \quad \delta_{1K_1}(X)
\end{array}
$$

$$\delta_{01}(X)\ \delta_{02}(X)\cdots\delta_{0K_0}(X)\quad \delta_{11}(X)\ \delta_{12}(X)\cdots\delta_{1K_1}(X)$$

$$\sum_{i=0,1}\sum_{j=1}^{K_i}\delta_{ij}(X)=1$$

In a random game, with probability $\delta_{ij}(X)$:
decide for Hypothesis $\mathbb{H}_i$ **and** subcase j.

We **SIMULTANEOUSLY** Detect and Isolate

Alternative two-step implementation

$$\underbrace{\delta_{01}(X), \delta_{02}(X), \ldots, \delta_{0K_0}(X)}_{+} \qquad \underbrace{\delta_{11}(X), \delta_{12}(X), \ldots, \delta_{1K_1}(X)}_{+}$$

$$\delta_0(X) \qquad\qquad \delta_1(X)$$

$$q_{ij}(X) = \frac{\delta_{ij}(X)}{\delta_i(X)}$$

$$\delta_0(X) + \delta_1(X) = 1$$

$$q_{01}(X) + \cdots + q_{0K_0}(X) = 1$$

$$q_{11}(X) + \cdots + q_{1K_1}(X) = 1$$

$$\delta_0(x), \delta_1(X), [q_{01}(X), \ldots, q_{0K_0}(X)], [q_{11}(X), \ldots, q_{1K_1}(X)]$$

- With probabilities $\delta_0(X)$,$\delta_1(X)$ decide between $\mathbb{H}_0$,$\mathbb{H}_1$.
- Given we selected $\mathbb{H}_i$, isolate among the subcases of $\mathbb{H}_i$ with probabilities $q_{i1}(X)$, $q_{i2}(X)$,..., $q_{iK_i}(X)$.

Optimality of GLRT

$$\begin{array}{l|l} \mathbb{H}_0 : X \sim f_{01}(X), \pi_{01} & \mathbb{H}_1 : X \sim f_{11}(X), \pi_{11} \\ \qquad\quad \sim f_{02}(X), \pi_{02} & \qquad\quad \sim f_{12}(X), \pi_{12} \\ \qquad\quad \vdots & \qquad\quad \vdots \\ \qquad\quad \sim f_{0K_0}(X), \pi_{0K_0} & \qquad\quad \sim f_{1K_1}(X), \pi_{1K_1} \end{array}$$

A Combined Detection / Isolation problem

Following a Neyman-Pearson like approach, we propose

$$\max \mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_1]$$

subject to the "false alarm" constraint

$$\mathbb{P}[\text{Miss-detection/isolation}|\mathbb{H}_0] \leq \alpha$$

THEOREM: The test that solves the combined detection/isolation problem is the following

$$\frac{\max\limits_{1\le j\le K_1} \pi_{1j} f_{1j}(X)}{\max\limits_{1\le j\le K_0} \pi_{0j} f_{0j}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

(Randomization probability γ)

Threshold λ and randomization probability γ are selected to satisfy the false alarm constraint with equality.

PROOF

$$\mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_i]$$

$$= \sum_{j=1}^{K_i} \mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_{ij}]\pi_{ij}$$

$$\mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_{ij}]$$

$$= \int \delta_i(X) q_{ij}(X) f_{ij}(X)\, dX.$$

Constrained optimization problem: Lagrange multiplier

$$\mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_1]$$

$$- \lambda \mathbb{P}[\text{Miss-detection/isolation}|\mathbb{H}_0]$$

$$= \mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_1]$$

$$+ \lambda \mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_0] - \lambda$$

$$\mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_1]$$
$$+ \lambda \mathbb{P}[\text{Correct-detection/isolation}|\mathbb{H}_0]$$

$$= \int \delta_1(X) \left\{ \sum_{j=1}^{K_1} q_{1j}(X) \pi_{1j} f_{1j}(X) \right\} dX$$
$$+ \lambda \int \delta_0(X) \left\{ \sum_{j=1}^{K_0} q_{0j}(X) \pi_{0j} f_{0j}(X) \right\} dX$$

$$\leq \int \delta_1(X) \max_{1 \leq j \leq K_1} \{\pi_{1j} f_{1j}(X)\} dX$$
$$+ \lambda \int \delta_0(X) \max_{1 \leq j \leq K_0} \{\pi_{0j} f_{0j}(X)\} dX$$

$$= \int \Big[\delta_1(X) \max_{1 \le j \le K_1} \{\pi_{1j} f_{1j}(X)\}$$

$$+ \delta_0(X) \lambda \max_{1 \le j \le K_0} \{\pi_{0j} f_{0j}(X)\} \Big] dX$$

$$\le \int \max \left\{ \max_{1 \le j \le K_1} \{\pi_{1j} f_{1j}(X)\}, \lambda \max_{1 \le j \le K_0} \{\pi_{0j} f_{0j}(X)\} \right\} dX$$

$$\frac{\max_{1 \le j \le K_1} \pi_{1j} f_{1j}(X)}{\max_{1 \le j \le K_0} \pi_{0j} f_{0j}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

If priors not known, assume equiprobable subcases:

$$\frac{\max_{1 \le j \le K_1} K_1^{-1} f_{1j}(X)}{\max_{1 \le j \le K_0} K_0^{-1} f_{0j}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda \Rightarrow \frac{\max_{1 \le j \le K_1} f_{1j}(X)}{\max_{1 \le j \le K_0} f_{0j}(X)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda \frac{K_1}{K_2}$$

Classical GLRT

Randomized estimators

Observations $X=[x_1,\ldots,x_N]$

$$X \sim f(X|\theta),\ \pi(\theta)$$

We would like to obtain an estimate $\hat{\theta}$ of θ

Deterministic estimator: $\hat{\theta} = G(X)$

Randomized estimator:

$$\delta_{\hat{\theta}}(X) \Rightarrow \delta(\hat{\theta}|X); \qquad \int \delta(\hat{\theta}|X)\, d\hat{\theta} = 1; \qquad \text{a PDF!}$$

How do we obtain a randomized estimate?

We generate a r.v. $\hat{\theta}$ distributed according to $\delta(\hat{\theta}|X)$

"Deterministic" estimator

$$\delta(\hat{\theta}|X) = \text{Dirac}\left\{\hat{\theta} - G(X)\right\}$$

Optimum Bayes estimation

If true parameter is θ and estimate $\hat{\theta}$, cost is: $\mathcal{C}(\hat{\theta}, \theta)$

Select randomized estimator to **minimize average cost**

$$\mathbb{E}[\mathcal{C}(\hat{\theta}, \theta)] = \iiint \mathcal{C}(\hat{\theta}, \theta)\delta(\hat{\theta}|X) f(X|\theta)\pi(\theta)\, dX\, d\hat{\theta}\, d\theta$$

$$= \int \left[\int \delta(\hat{\theta}|X) \left\{ \underbrace{\int \mathcal{C}(\hat{\theta}, \theta) f(X|\theta)\pi(\theta)\, d\theta}_{\mathcal{D}(\hat{\theta}, X) \geq \min_U \mathcal{D}(U, X)} \right\} d\hat{\theta} \right] dX$$

$$\geq \int \min_U \mathcal{D}(U, X)\, dX$$

$$\delta(\hat{\theta}|X) = \text{Dirac}\left\{ \hat{\theta} - \arg\min_U \mathcal{D}(U, X) \right\}$$

Combined detection/estimation

Observations $X=[x_1,\ldots,x_N]$

$$\mathbb{H}_0 : X \sim f(X|\theta=0)$$

$$\mathbb{H}_1 : X \sim f(X|\theta),\ \pi(\theta)$$

- Decide between $\mathbb{H}_0,\mathbb{H}_1$
- Whenever we decide in favor of $\mathbb{H}_1$ we also like to provide an estimate $\hat{\theta}$ of θ

Detection/estimation procedure: Two-step strategy

$$\delta_0(X), \delta_1(X), q(\hat{\theta}|X)$$

Following a Neyman-Pearson like approach, we propose

$$\min \mathbb{E}[\mathcal{C}(\hat{\theta}, \theta)|\mathbb{H}_1]$$

subject to: $\mathbb{P}[\text{Miss-detection}|\mathbb{H}_0] \leq \alpha$

$$\mathbb{P}[\text{Miss-detection}|\mathbb{H}_0] = \int \delta_1(X) f(X|0) dX$$

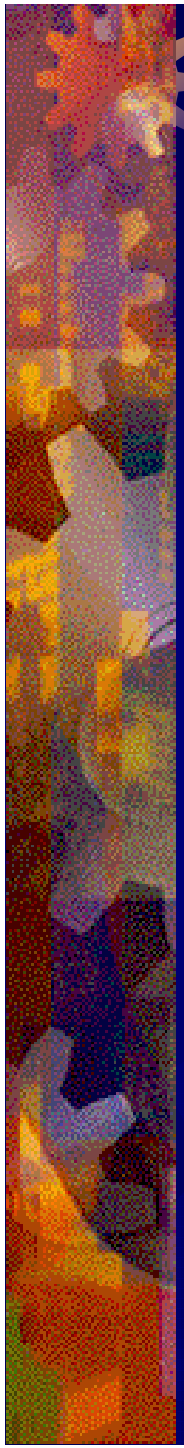
Cost computation

Correct Hypothesis:

$$\int\int\int \mathcal{C}(\hat{\theta}, \theta) \delta_1(X) q(\hat{\theta}|X) \pi(\theta) f(X|\theta) d\theta \, d\hat{\theta} \, dX$$

Wrong Hypothesis:

$$\int\int \mathcal{C}(0, \theta) \delta_0(X) \pi(\theta) f(X|\theta) d\theta \, dX$$



$$\iiint \mathcal{C}(\hat{\theta},\theta)\delta_1(X)q(\hat{\theta}|X)\pi(\theta)f(X|\theta)d\theta\, d\hat{\theta}\, dX$$

$$+ \iint \mathcal{C}(0,\theta)\delta_0(X)\pi(\theta)f(X|\theta)d\theta\, dX$$

$$+ \lambda \int \delta_1(X)f(X|0)dX$$

$$\geq \int \delta_1(X) \min_U \overbrace{\left\{ \int \mathcal{C}(U,\theta)\pi(\theta)f(X|\theta)d\theta \right\}}^{\mathcal{D}(U,X)} dX$$

$$+ \int \delta_0(X) \underbrace{\left\{ \int \mathcal{C}(0,\theta)\pi(\theta)f(X|\theta)d\theta \right\}}_{\mathcal{D}(0,X)} dX$$

$$+ \lambda \int \delta_1(X)f(X|0)dX$$

$$= \int \Big\{ \delta_1(X)[\lambda f(X|0) + \min_U \mathcal{D}(U,X)] + \delta_0(X)\mathcal{D}(0,X) \Big\} dX$$

$$\geq \int \min \Big\{ \lambda f(X|0) + \min_U \mathcal{D}(U,X), \mathcal{D}(0,X) \Big\} dX$$

We decide in favor of $\mathbb{H}_1$ when

$$\mathcal{D}(0,X) > \lambda f(X|0) + \min_U \mathcal{D}(U,X)$$

The final test is: $$\frac{\mathcal{D}(0,X) - \min_U \mathcal{D}(U,X)}{f(X|0)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

$$\mathcal{C}(U,X) = \int \mathcal{C}(U,\theta) f(X|\theta) \pi(\theta) d\theta$$

Examples

MAP estimator:

$$\mathcal{C}(\hat{\theta},\theta) = \begin{cases} 0 & \text{for } \|\hat{\theta}-\theta\| \leq \Delta \ll 1 \\ 1 & \text{otherwise} \end{cases}$$

Optimum test:

$$\frac{\max_{\theta} \pi(\theta) f(X|\theta)}{f(X|0)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

If prior $\pi(\theta)$ is uniform we recover the classical GLRT

$$\frac{\max_{\theta} f(X|\theta)}{f(X|0)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

MMSE estimator:

$$\mathcal{C}(\hat{\theta}, \theta) = \|\hat{\theta} - \theta\|^2$$

Optimum test:

$$\|\hat{\theta}_{\mathrm{MMSE}}\|^2 \frac{\int \pi(\theta) f(X|\theta)\, d\theta}{f(X|0)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

$$\hat{\theta}_{\mathrm{MMSE}} = \frac{\int \theta \pi(\theta) f(X|\theta)\, d\theta}{\int \pi(\theta) f(X|\theta)\, d\theta} = \mathbb{E}[\theta|X]$$

MEDIAN estimator:

$$\mathcal{C}(\hat{\theta}, \theta) = |\hat{\theta} - \theta|$$

Optimum test:

$$\frac{\displaystyle\int_0^{\hat{\theta}_{\text{MEDIAN}}} \theta \pi(\theta) f(X|\theta)\, d\theta}{f(X|0)} \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda$$

$$\hat{\theta}_{\text{MEDIAN}} = \arg \left\{ \hat{\theta} : \frac{\displaystyle\int_{-\infty}^{\hat{\theta}} \pi(\theta) f(X|\theta)\, d\theta}{\displaystyle\int_{-\infty}^{\infty} \pi(\theta) f(X|\theta)\, d\theta} = 0.5 \right\}$$

$$= \arg\{\hat{\theta} : \mathbb{P}[\theta \leq \hat{\theta}|X] = 0.5\}$$

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225 North Ave.
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Electrical Engineering Seminar

"Finite Sample Size Optimality of GLR Tests"



Professor George V. Moustakides,
Department of Electrical & Computer Engineering, University of Patras

Time: 11:00 a.m. Tuesday September 2, 2008

Place: EE Conference Room (MUDD 1306)

Abstract

In binary hypothesis testing, when hypotheses are composite or the corresponding data pdfs contain unknown parameters, the generalized likelihood ratio test (GLRT) constitutes a popular means for deciding between the two possibilities. GLRT has the very interesting characteristic of performing simultaneous detection and estimation in the case of parameterized pdfs or combined detection and isolation in the case of composite hypotheses. Even though this test is known for years and has been the decision tool in numerous applications, existing results demonstrate only large sample size asymptotic optimality. In our presentation we introduce a novel, finite sample size detection/estimation formulation for the problem of hypothesis testing with unknown parameters and a corresponding detection/isolation setup for the case of composite hypotheses. The optimum test that results from our performance measure optimization has a GLRT-like structure which is closely related to the criterion we employ for the parameter estimation or isolation part. When this criterion is selected in a very specific manner, we recover the classical GLRT of the literature, while we obtain interesting novel tests with alternative criteria. Our mathematical derivations are surprisingly simple considering that they solve a problem that has been open for more than half a century.

Bio

George V. Moustakides was born in Greece. He obtained the diploma in Electrical and Mechanical Engineering from the National Technical University of Athens in 1978, the MSc in Systems Engineering from UPENN in 1980 and the PhD in Electrical Engineering and Computer Science from Princeton University in 1983. From 1983 to 1986 he held a research position with INRIA, France and from 1988 to 1991 a research position with the Computer Technology Institute of Patras, Greece. Since 1991 he has been a faculty member with the University of Patras, Greece. His research focuses on various problems in Statistical Signal Processing, Sequential Detection and Signal Processing for Hearing Aids.

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Finite Sample Size Optimality of GLR Tests

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EVENT CONTACT:

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TITLE: Finite Sample Size Optimality of GLR Tests

SPEAKER: Dr. George **Moustakides**

ABSTRACT:

In binary hypothesis testing, when hypotheses are composite or the corresponding data pdfs contain unknown parameters, the generalized likelihood ratio test (GLRT) constitutes a popular means for deciding between the two possibilities. GLRT has the very interesting characteristic of performing simultaneous detection and estimation in the case of parameterized pdfs or combined detection and isolation in the case of composite hypotheses. Even though this test is known for years and has been the decision tool in numerous applications, existing results demonstrate only large sample size asymptotic optimality.

In our presentation we introduce a novel, finite sample size detection/estimation formulation for the problem of hypothesis testing with unknown parameters and a corresponding detection/isolation setup for the case of composite hypotheses. The optimum test that results from our performance measure optimization has a GLRT-like structure which is closely related to the criterion we employ for the parameter estimation or isolation part. When this criterion is selected in a very specific manner, we recover the classical GLRT of the literature, while we obtain interesting novel tests with alternative criteria. Our mathematical derivations are surprisingly simple considering that they solve a problem that has been open for more than half a century.